November 21, 2009
Bethel College

Blaise Pascal Mathematics Competition

Directions:

- Do not open this booklet until you are instructed to do so.
- This exam contains 45 multiple choice questions, each with five possible answers. Only one answer is the correct answer.
- Please record your answers on the given **scantron sheet** using only spaces 1 through 45. Use a No. 2 pencil (either the one provided or your own) for marking your answers. Do not make any extraneous marks on your scantron sheet, as this will affect the scoring. Be sure that you completely fill in the bubble, and erase your mistakes fully.
- Scoring will be as follows: You will receive 1 point for a correct answer, zero points for an answer left blank, and $-\frac{1}{4}$ for an incorrect answer. Thus, it is not to your advantage to guess.
- You will have 90 minutes to complete the exam.
- If your booklet is defective or has a missing page, please notify someone immediately.
- Drawings on the exam are not to scale.
- No external aids may be utilized during the exam except a pencil and scratch paper. In particular, no calculators will be allowed during the exam.
- When time is called, you must stop working and turn in your scantron form. The paper copy of the exam is yours to keep, as is the pencil. You may also collect an answer sheet on your way out.

1. How many terms after the decimal place will be in the decimal expansion of $\frac{2009}{20000}$?

   (a) 5 (b) 6 (c) 7 (d) 8 (e) 9

2. Working alone, Jane can mow the lawn in 1.5 hours, and Bill can do it in 3 hours. How many hours will it take them if they work together, each with their own mower?

   (a) 0.25 (b) 0.5 (c) 0.75 (d) 1 (e) 1.25

3. Let $f(n) = n^2 - 19n + 99$ where $n = 1, 2, 3, \ldots$. For how many values of n is $f(n)$ a perfect square?

   (a) 2 (b) 3 (c) 4 (d) 5 (e) more than 5

4. Suppose that a, b, and c are values for which the number $708, a6b, 8c9$ is a multiple of 9. Which of the following is a possible value of $a + b + c$?

   (a) 6 (b) 8 (c) 14 (d) 16 (e) 22

5. How many positive multiples of 10 that are less than 1000 are the sum of four consecutive integers?

   (a) 45 (b) 47 (c) 50 (d) 52 (e) none of these

6. How many distinguishable ways can the letters in the word PASCAL be rearranged? (Note that switching the P and the S yields a distinguishable rearrangement, but simply switching the two A's does not.)

   (a) 60 (b) 360 (c) 450 (d) 600 (e) 720

7. How many rational roots does the polynomial $g(x) = 2x^4 + 4x^2 + 8$ have?

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

8. How many zeros are at the end of 2009!, where $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1$?

   (a) 100 (b) 200 (c) 300 (d) 400 (e) 500

9. Pascal and his friend each have a pair of dice that they are going to roll. Which of the following is *least* likely to occur when they roll?

   (a) Pascal gets a sum of 6 (b) Pascal's friend gets a sum of 7
   (c) Pascal and his friend get the same sum
   (d) Pascal gets a sum less than 6 and his friend gets a sum greater than 8
   (e) Pascal gets a sum of either 4 or 10

10. Let a_n be a sequence in which $a_1 = 2009$ and for $n \geq 1$ we define $a_{n+1} = 1 + \frac{1}{a_n}$. Which of the following is closest to a_{2009}?

    (a) -0.5 (b) 0 (c) 1 (d) 1.5 (e) 2

11. What is the ones digit of $2009^{2009} + 2010^{2009} + 2011^{2009} + 2012^{2009}$?

    (a) 0 (b) 2 (c) 4 (d) 6 (e) 8

12. How many solutions does the equation $\log_x 4 = 2$ have?

    (a) 0 (b) 1 (c) 2 (d) 3 (e) more than 3

13. A circle of radius one and center $(1, 1)$ is drawn in the plane. A second circle is drawn that is tangent to the x- and y-axes as well as externally tangent to the original circle, as shown at right. Find the radius of the smaller circle.

    (a) $\frac{1}{\sqrt{5}}$ (b) $3 - 2\sqrt{2}$ (c) $\sqrt{2} - 1$ (d) $\frac{1}{5}$ (e) none of these

14. Suppose that the operation $\square$ is defined by $a \square b = a + b - ab$ (where a and b are integers). Which of the following properties hold?

I. $a \square b = b \square a$

II. $(a \square b) \square c = a \square (b \square c)$

III. $a \square 0 = 0$

(a) I only (b) II only (c) I and II only
(d) I and III only (e) I, II, and III

15. A parallelogram with side lengths 4 and 5 has a diagonal of length 8. How long is the other diagonal?

(a) 3 (b) $3\sqrt{2}/2$ (c) 4 (d) $\sqrt{41}$ (e) none of these

16. The function R reverses the digits of a base ten number. (For instance, $R(123) = 321$.) We say that a natural number is a **palindrome** if $R(n) = n$. What percent of all four-digit numbers are not palindromes? (Note: The number 0123 is not a four-digit number.)

(a) 97% (b) 98% (c) 99% (d) 99.5% (e) 99.9%

17. The midpoints of the longer sides of a rectangle with side lengths 2 and 4 are joined to opposite vertices as shown at right. Find the area of the shaded quadrilateral.

(a) 2 (b) $\sqrt{2}$ (c) $2\sqrt{2}$ (d) 1 (e) none of these

18. If $x^4 + x^2 + 1 = 0$, what is the value of $(x^2 + \frac{1}{x^2})^3$?

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

19. Find the sum of all real numbers x such that $|x - 2008| + |x - 2009| = 3$.

(a) 2009 (b) 4017 (c) 6025 (d) 8034 (e) none of these

20. Which of the following is equal to $0.000037\overline{000037}$?

(a) $\frac{1}{27,027}$ (b) $\frac{37}{999990}$ (c) $\frac{37}{100000}$ (d) $\frac{1}{3737}$ (e) none of these

21. Find the length of the longest rod that can be placed in a box with side lengths of 3, 4, and 5.

(a) 6 (b) 12 (c) $5\sqrt{2}$ (d) $5\sqrt{3}$ (e) $3\sqrt{5}$

22. How many points do the graphs of $|x| + |y| = 4$ and $x^2 + 16y^2 = 16$ have in common?

(a) 2 (b) 4 (c) 6 (d) 8 (e) 10

23. Pascal's Pizza Shop has six different toppings to choose from for their pizzas. A customer may choose any number of toppings on a pizza, but no repeated toppings are allowed. If a customer chooses n toppings for their pizza, which value of n gives the largest number of choices for the customer?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

24. How many digits are in the number generated by $2^{38}(25)^{16}$?

(a) 30 (b) 31 (c) 32 (d) 33 (e) 34

25. Suppose that f and g are two real-valued functions such that $f(g(x)) = g(f(x)) = x$ and $f(x) = 3x + 4$. Find the y-coordinate of the point(s) where the graphs of f and g intersect (if they intersect).

(a) $y = -2$ (b) $y = 0, -2$ (c) $y = 0, 2$ (d) $y = 2$
(e) f and g do not intersect

26. What is the least common multiple of 6027 and 10045?

(a) 20090 (b) 30135 (c) 40180 (d) 50225 (e) 60270

27. Euler's Donut Shop sells three kinds of donuts: glazed, powdered, and creme-filled. You need to buy three donuts, but you want no more than 2 glazed donuts. How many ways can you do this?

(a) 1 (b) 2 (c) 5 (d) 9 (e) 14

28. You and two of your friends each have a key corresponding to a different locker. (Thus, there are a total of three keys to three different lockers.) A bully comes up, grabs all three keys, mixes them up, randomly throws one in each locker, and slams all three lockers shut. The principal comes and pries open one of the lockers, thus obtaining the key inside. What is the probability that you will then be able to open the two other lockers without having to pry them open?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$ (e) 1

29. Let $P(x) = 41x^2 + ax + 2009$. For which value of a will $P(x) = 0$ have exactly one solution?

(a) 374 (b) 474 (c) 574 (d) 674 (e) there is no such value

30. Let $Q(x)$ be a polynomial with integer coefficients such that $Q(1) = Q(2) = Q(3) = 2009$. How many possible integer solutions x are there to $Q(x) = 2010$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

31. An automobile traveled 6 hours at an average speed of 40 miles per hour. It averaged only 30 miles per hour on the return trip. Which of the following is closest to the average speed in miles per hour for the round trip?

(a) 33 (b) 34 (c) 35 (d) 36 (e) 37

32. Define $f_1(x) = \frac{x}{x+1}$, and for $n \geq 1$, define $f_{n+1}(x) = f_n(f_1(x))$. Determine $f_{2009}(1)$.

(a) $\frac{2008}{2009}$ (b) $\frac{2009}{2010}$ (c) $\frac{1}{2009}$ (d) $\frac{1}{2010}$ (e) none of these

33. $\arcsin\frac{1}{5} + \arccos\frac{1}{5} + \arctan\frac{1}{5} + \operatorname{arccot}\frac{1}{5} + \operatorname{arcsec}\frac{1}{5} + \operatorname{arccsc}\frac{1}{5} =$

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) $\frac{3\pi}{2}$ (e) 2π

34. A cone is constructed from a sector that is one seventh of a circle of radius 2009 by gluing the sector along the two outer radii, as shown at right. What will be the radius of the base of the resulting cone?

(a) 7 (b) 49 (c) 287 (d) 343 (e) 2009

35. A rectangle is constructed with base 4 and height 1. A semicircle is constructed with a diameter corresponding to the base of the rectangle, as shown at right. What is the area of the shaded region?

(a) $\frac{4\pi}{3} - \sqrt{3}$ (b) $\frac{4\pi}{3}$ (c) $\frac{5\pi}{3}$ (d) $\frac{2\sqrt{3}}{3}\pi$ (e) $\sqrt{5} - \frac{2\pi}{3}$

36. In the circle pictured at right, $AB = 4$, $AP = 4$, and $PC = 3$. Find the length of CD.

(a) 3 (b) 4 (c) 5 (d) 6 (e) none of these

37. Which of the following is closest to the value of the sum

$$\log_{10} 1 + \log_{10} 2 + \log_{10} 3 + \cdots + \log 1010$$

(a) 2 (b) 4 (c) 6 (d) 8 (e) 10

38. The sum

$$\frac{1+3+5+\cdots+2009}{2010}$$

is closest to which of the following?

(a) 500 (b) 550 (c) 600 (d) 650 (e) 700

39. If $(x^4-16)(x^2-4x+3)=(x^2-5x+6)h(x)$, what is $h(2)$?

(a) 0 (b) 8 (c) 16 (d) 18 (e) 32

40. Train A leaves at noon from Hickville traveling east, while Train B leaves at noon from Smallville traveling west. Smallville is 60 miles east of Hickville. Both trains travel at 10 mph. A bird also leaves at noon from Smallville flying at 20 mph west in front of Train B. When the bird reaches Train A, it turns around and flies east (at 20 mph). When the bird reaches Train B, it again turns around and flies west. The bird continues to do this until the two trains meet between Hickville and Smallville. How far did the bird travel during this time?

(a) 30 miles (b) 40 miles (c) 50 miles (d) 60 miles (e) 70 miles

41. Which civilization is commonly attributed with the initial development of algebra?

(a) the Mayans (b) the Persians (c) the Europeans
(d) the Greeks (e) the Chinese

42. In the year 2000, how many days did February have?

(a) 27 (b) 28 (c) 29 (d) 30 (e) 31

43. In 1897 the Indiana legislature attempted to pass a bill that implied that π is equal to what value?

(a) 3 (b) 3.14 (c) 3.2 (d) 3.3 (e) 4

44. Which of the following mathematicians lived at the same time as Blaise Pascal?

(a) Rene Descartes (b) Sir Isaac Newton (c) Gottfried Wilhelm Leibniz
(d) all of them lived at the same time as Pascal
(e) none of them lived at the same time as Pascal

45. In Pascal's famous triangle, what line comes after "1 4 6 4 1"?

(a) 1 4 7 4 1 (b) 1 5 6 5 1 (c) 1 5 7 5 1 (d) 1 5 10 5 1 (e) 1 5 10 10 5 1